Decision making by fuzzy soft sets with an application

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ABSTRACT. In this paper we define positive and negative parameters with various degrees of importance. Then, we present a novel method of object recognition from an imprecise data set. The method involves construction of a Comparison Table from a fuzzy soft set with mentioned parameters for decision making. An application of this method is also illustrated with an example.

Keywords: fuzzy soft set, positive parameter, negative parameter.

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1. Introduction

Many of real life problems in social and medical sciences, engineering, economics etc. involve imprecise data. The solution of these problems requires the use of mathematical methods that based on imprecision and uncertainty. In recent years, researchers have been proposed a number of theories for dealing with this problems in an effective way, such as fuzzy set theory [7, 8], theory of probability, intuitionistic fuzzy sets [1] , vague sets [2], rough set theory [5], theory of interval mathematics [1] etc. A novel concept of soft theory initiated by Molodtsov as a new mathematical tool for dealing with uncertainties. The soft set introduced by Molodtsov [4], etc. is a set associated with a set of parameters and has been applied in several ways. In this paper, we present an application of fuzzy soft sets in decision making problem. This paper organized as follow. In section 1, we recall the relevant definitions and results of soft set theory and fuzzy set theory. Then we define the positive (negative) parameter as parameter that we aspire to have a high (low) degree of membership. Then, we define a degree of importance (weight) for each parameter. Also, we presented an algorithm for decision making problems on fuzzy soft set with mentioned parameters. An application of this algorithm has been presented in section 2, to choose the best vaccine from the fuzzy set including five vaccines.

1.1. Preliminaries. In this section, first we collect the relevant definitions and results of soft set theory and fuzzy set theory make this paper self-contained and easy to read. Definitions and results presented in this section may be found in [7, 8].

DEFINITION 1.1. Let U be an initial universe set and E be a set of parameters. Let $P(U)$ denote the power set of U and $A \subseteq E$. A pair (F, A) is called a soft set over U, where F is a mapping given by $F : A \to P(U)$. In other words, a soft set over U is a parametrized family of subsets of the universe U. For $\alpha \in A, F(\alpha)$ may be considered as the set of -approximate elements of the soft set (F, A).

DEFINITION 1.2. For two soft sets (F, A) and (G, B) over a common universe U, we say that (F, A) is a soft subset of (G, B) if (i) $A \subseteq B$, and (ii) $\forall \alpha \in A, F(\alpha)$ and $G(\alpha)$ are identical approximations. We write $(F, A)\tilde{\subset}(G, B)$.

DEFINITION 1.3. If (F, A) and (G, B) be two soft sets then "$(F, A)AND(G, B)$" denoted by $(F, A) \wedge (G, B)$ is defined by $(F, A) \wedge (G, B) = (H, A \times B)$, where $H(\alpha, \beta) = F(\alpha) \wedge G(\beta), \forall(\alpha, \beta) \in A \times B$.

DEFINITION 1.4. If (F, A) and (G, B) be two soft sets then "$(F, A)OR(G, B)$" denoted by $(F, A) \vee (G, B)$ is defined by $(F, A) \vee (G, B) = (H, A \times B)$, where $H(\alpha, \beta) = F(\alpha) \vee G(\beta), \forall(\alpha, \beta) \in A \times B$.

1.2. decision making by fuzzy soft sets. In this section we recall fuzzy soft set and some results of it [3, 6]. Let $U = o_1, o_2, ..., o_k$ be a set of K objects, which specified by a set of parameters $A_1, A_2, ..., A_i$. Let E the parameter space and $\forall i, A_i \subseteq E$. Also, any A_i represents a specific property set. Here we assume that these property sets are fuzzy sets.

DEFINITION 1.5. For two soft sets (F, A) and (G, B) over a common universe U, we say that (F, A) is a fuzzy soft subset of (G, B) if $A \subseteq B$, and $\forall \alpha \in A, F(\alpha)\tilde{\subset}G(\alpha)$.

DEFINITION 1.6. (Zadeh 1965) Consider a universal set V and the membership function $\mu : V \to [0, 1]$. Then, V_μ is fuzzy set on V if each element $\alpha \in V$ is associated with degree of membership, which is a real number in $[0, 1]$ and it is denoted by μ_α

DEFINITION 1.7. Let α be a parameter.
(i) We say α is a *positive parameter* if we are interested in high degree of α.
(ii) We say α is a *negativee parameter* if we are interested in low degree of α.
(iii) We show the importance degree the parameter α by λ_α, where λ_α is a positive integer from 1 to 10

DEFINITION 1.8. If (F, A) and (G, B) be two soft sets then "$(F, A)AND(G, B)$" denoted by $(F, A) \wedge (G, B)$ is defined by $(F, A) \wedge (G, B) = (H, A \times B)$, where $H(\alpha, \beta) = F(\alpha)\tilde{\cap}G(\beta), \forall(\alpha, \beta) \in A \times B$.
(i) If α and β are positive parameters $F(\alpha)\tilde{\cap}G(\beta) = min(\lambda_\alpha \times f_\alpha, \lambda_\beta \times f_\beta)$.
(ii) If α and β are negative parameters $F(\alpha)\tilde{\cap}G(\beta) = min(\lambda_\alpha \times 1 - \mu_\alpha, \lambda_\beta \times 1 - \mu_\beta)$.
(iii) If α is a positive parameter and β is a negative parameter $F(\alpha)\tilde{\cap}G(\beta) = min(\lambda_\alpha \times \mu_\alpha, \lambda_\beta \times 1 - \mu_\beta)$.

Comparison table is a square table in which the number of rows and number of columns are equal, rows and columns both are labelled by the object names $o_1, o_2, o_3, ..., O_n$ on of the universe, and the entries are $c_{ij}, i, j = 1, 2, ..., n$. c_{ij} is the number of positive parameters for which the product of membership value and importance degree of o_i exceeds or equal to the product of membership value and importance degree of o_j plus the number of negative parameters that the product of membership value and importance degree of o_i exceeds or equal to the product of (1-membership value) and importance degree of o_j. It is clear that, $0 \leq C_{ij} \leq n$, and $c_{ii} = n, \forall i, j$ where, n is the number of parameters present in a fuzzy soft set. The row(column) sum of an object o_i is denoted by $r_i(l_i)$ and is calculated by using the formula,

$$r_i = \sum_{j=1}^{n} c_{ij} \quad (l_i = \sum_{j=1}^{n} c_{ij})$$

The score of an object o_i is S_i is given as $S_i = r_i - l_i$. We are going to choose an object from the set of given objects with respect to a set of choice parameters. To do this, we present an algorithm.

1.3. Algorithm.

(1) Input fuzzy-soft-sets (F, A) and (G, B).
(2) Input the parameter set (P) as observed by the expert.
(3) Input the importance degree λ_i for any p_i.
(4) Put $P^- = \{p \in P : p \; is \; a \; positive \; parameter\}$.
(5) Put $P^+ = \{p \in P : p \; is \; a \; negative \; parameter\}$.
(6) Compute the corresponding resultant-fuzzy-soft-set (S, P) from the fuzzy soft sets (F, A) and (G, B).
(7) Construct the Comparison-table of the fuzzy-soft-set (S, P) and compute r_i and l_i for $o_i, \forall i$.
(8) Compute the score of $o_i, \forall i$.
(9) The decision is S_k if, $S_k = maxi \; S_i$.
(10) If k has more than one value then any one of o_k may be chosen.

2. Application decision making for choosing an vaccine

In this section, we present an example for application of the aforementioned algorithm.

Example 2.1. Let $U = \{V_1, V_2, V_3, V_4, V_5\}$ be the set of different vaccines. The parameter set with importance degrees as follow:

$$\begin{aligned} P =&\{(Reduce \; mortality, 10), (Reduce \; infection, 8), \\ &(Reduction \; of \; hospitalization, 9), (Side \; of \; effects, 8), (Cost, 4)\}, \end{aligned}$$

We now

$P^+ = \{(Reduce \; mortality, 10), (Reduce \; infection, 8), (Reduction \; of \; hospitalization, 9)\}$

$P^- = \{(side \; of \; effects, 7), (cost, 4)\}$

We consider

$a_1 = Reduce \; mortality, a_2 = Reduce \; infection, \; a_3 = Reduction \; of \; hospitalization,$

$a_4 = side \; of \; effects, \; a_5 = Cost.$

Let the fuzzy soft set be computed as bellow:

U	a_1	a_2	a_3	a_4	a_5
V_1	0.9	0.8	0.7	0.02	0.5
V_2	0.8	0.7	0.6	0.04	0.6
V_3	0.6	0.5	0.3	0.02	0.7
V_4	0.9	0.7	0.5	0.01	0.4
V_5	0.7	0.6	0.4	0.03	0.5

The following table is the fuzzy soft set with considering the negative and positive parameters.

U	a_1	a_2	a_3	a_4	a_5
V_1	0.9	0.8	0.7	0.98	0.5
V_2	0.8	0.7	0.6	0.96	0.4
V_3	0.6	0.5	0.3	0.98	0.3
V_4	0.9	0.7	0.5	0.99	0.6
V_5	0.7	0.6	0.4	0.97	0.5

Now we compute soft set with importance degrees

U	a_1	a_2	a_3	a_4	a_5
V_1	9	6.4	6.3	7.84	2
V_2	8	5.6	5.4	5.28	1.4
V_3	6	4.0	2.7	7.84	1.2
V_4	9	5.6	5.4	7.92	2.4
V_5	7	4.8	6.4	7.76	2

The Comparison-Table of the above soft set is as below.

U	V_1	V_2	V_3	V_4	V_5
V_1	5	5	5	3	4
V_2	0	5	4	2	2
V_3	1	1	5	0	2
V_4	3	5	5	5	3
V_5	2	2	4	1	5

Next we compute the row-sum, column-sum, and the score for each V_i as shown below:

U	$row - sum(r_i)$	$column - sum(l_i)$	$score$
V_1	22	11	11
V_2	13	18	-5
V_3	9	23	-14
V_4	21	11	10
V_5	14	16	-2

As we expected,considering the table above, the best vaccine is V_1.

2.1. Conclusion. The soft set theory was originated by Molodtsov as a general mathematical tool for dealing with fuzzy,uncertain, or vague objects. In this paper we give an application of fuzzy soft set theory in in decision making problems. The recognition strategy is based on fuzzy input parameter data set. In this method involves the positive and negative parameters with different weights. The maximum score in Comparison Table is the best decision.

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